18. a) Three quantities P, Q and R are such that P varies directly as Q and inversely as the square root of R. Given that P = 2250 when Q = 450 and R = 64, write down an equation connecting P, Q and R. (4 marks)

$P \propto \frac{Q}{\sqrt{R}}$

$P = \frac{KQ}{\sqrt{R}}$

$2250 = \frac{450K}{\sqrt{64}}$

$2250 = \frac{450K}{8}$

$\frac{450K}{450} = \frac{2250 \times 8}{450}$

$K = \frac{18000}{450}$

$= 40$

$P = \frac{40Q}{\sqrt{R}}$

(i) If Q is decreased by 16% and R increased by 44%, calculate the percentage change in P. (3 mks)

$P_1 = \frac{40 \times 0.84Q}{\sqrt{1.44R}}$

$= \frac{33.6Q}{1.2\sqrt{R}}$

$= \frac{28Q}{\sqrt{R}}$

$\%\text{Change} = \frac{P_1 - P}{P} \times 100$

$= \frac{\frac{28Q}{\sqrt{R}} - \frac{40Q}{\sqrt{R}}}{40\frac{Q}{\sqrt{R}}} \times 100$

$= \frac{\frac{Q}{\sqrt{R}}(28-40)}{40\frac{Q}{\sqrt{R}}} \times 100$

$= \frac{28-40}{40} \times 100$

$= \frac{-12}{40} \times 100$

$= -30$

= Percentage change in P is decreased by 30%

(b) In a soccer competition, the number of goals (G) scored in a penalty shoot — out is partly constant and partly varies as the skill (S) of the player. Given that G = 8 when S = 2 and G = 12 when S = 4, find the value of G when S 6. (3 marks)

$G = C + KS$

$8 = C + 2K \text{ ......(i)}$ } Subtract

$12 = C + 4K \text{ .....(ii)}$

$\frac{-4}{-2} = \frac{-2K}{-2}$

$K = 2$

$\Rightarrow 8 = C + 4$

$C = 8 - 4$

$= 4$

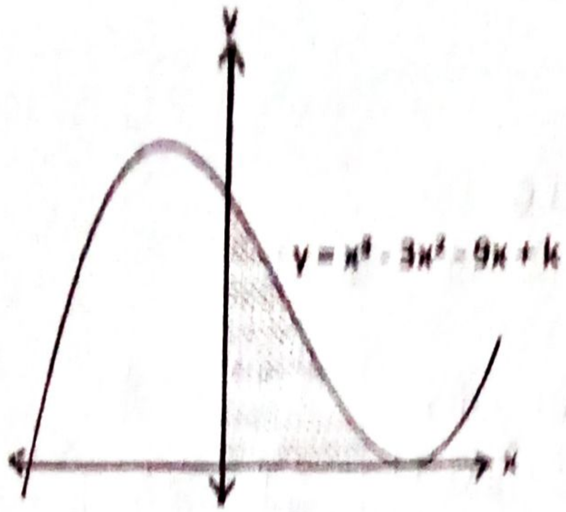
$G = 4 + 2S$

When $S = 6$

$G = 4 + 12$

$G = 16$

20. The diagram below shows the curve $y = x^3 - 3x^2 - 9x + k$, where k is a constant. The curve has a minimum point on the x-axis.



a) Find the value of k (5mks)

At turning point $\frac{dy}{dx} = 0$

$0 = 3x^2 - 6x - 9$

$x = \frac{6 \pm \sqrt{36 + 108}}{6}$

$x = \frac{6 \pm 12}{6}$

$x = \frac{18}{6}$ or $\frac{-6}{6}$

$x = 3$ or -1

$x = 3$

Min point = $(3, 0)$

$0 = 3^3 - 3(3)^2 - 9(3) + k$

$0 = 27 - 27 - 27 + k$

$27 = K$

b) The coordinates of the maximum point of the curve. (2mks)

Max point $x = 1$

$y = (-1)^3 - 3(-1)^2 - 9(-1) + 27$

$y = 32$

Coordinates of max point $= (-1, 32)$

c) Find the area of the shaded region. (3 mks)

$\int_0^3 (x^3 - 3x^2 - 9x + 27)\,dx$

$= \left[\frac{x^4}{4} - \frac{3x^3}{3} - \frac{9x^2}{2} + 27x\right]_0^3$

$= \left[\frac{3^4}{4} - \frac{3(3)^3}{3} - \frac{9(3)^2}{2} + 27(3)\right] - 0$

$= \frac{81}{4} - 27 - \frac{81}{2} + 81$

$= \frac{135}{4}$ square units

or

33.75 square units

21. OPQ is a triangle in which OP=P and OQ=q. x is a point on OP such that OP:XP=5:2 and y is another point on PQ such that PY:YQ=1:2. Lines OY and XQ intersect at T.

(a) Express the following vectors in terms of P and q

(i) PQ (1 mark)

$\overline{PQ} = \overline{PO} + \overline{OQ}$

$= -\underset{\sim}{p} + \underset{\sim}{q} = \underset{\sim}{q} - \underset{\sim}{p}$

(ii) OY (1 mark)

$\overline{OY} = \overline{OP} + \overline{PY}$

$= \underset{\sim}{p} + \frac{1}{3}(\underset{\sim}{q} - \underset{\sim}{p})$

$= \underset{\sim}{p} + \frac{1}{3}\underset{\sim}{q} - \frac{1}{3}\underset{\sim}{p}$

$= \frac{2}{3}\underset{\sim}{p} + \frac{1}{3}\underset{\sim}{q}$

(iii) QX (1 mark)

$QX = \overline{QO} + \overline{OX}$

$= -\underset{\sim}{q} + \frac{3}{5}\underset{\sim}{p}$

$= \frac{3}{5}\underset{\sim}{p} - \underset{\sim}{q}$

(b) If $OT = kOY$ and $QT = hQX$ express OT in two different ways. Hence or otherwise find the values of h and k. (6 marks)

$\overline{OT} = k(\overline{OY})$

$= k\left(\frac{2}{3}\underset{\sim}{p} + \frac{1}{3}\underset{\sim}{q}\right)$

$\overline{OT} = \frac{2}{3}k\underset{\sim}{p} + \frac{1}{3}k\underset{\sim}{q}$ ----(i)

$\overline{OT} = \overline{OQ} + \overline{QT}$

$= \underset{\sim}{q} + h(Qx)$

$= \underset{\sim}{q} + h\left(\frac{3}{5}\underset{\sim}{p} - \underset{\sim}{q}\right)$

$= \underset{\sim}{q} + \frac{3}{5}h\underset{\sim}{p} - h\underset{\sim}{q}$

$= \underset{\sim}{q}(1-h) + \frac{3}{5}h\underset{\sim}{p}$ ----(ii)

$= \frac{2}{3}k\underset{\sim}{p} + \frac{1}{3}k\underset{\sim}{q} = \underset{\sim}{q}(1-h) + \frac{3}{5}h\underset{\sim}{p}$

$= \frac{2}{3}k\underset{\sim}{p} = \frac{3}{5}h\underset{\sim}{p}$

$\frac{2}{3}k = \frac{3}{5}h$

$K = \frac{3}{5}h \times \frac{3}{2} = \frac{9}{10}h$

and

$\frac{1}{3}k\underset{\sim}{q} = \underset{\sim}{q}(1-h)$

$\Rightarrow \frac{1}{3}\left(\frac{9}{10}h\right) = 1-h$

$\frac{9}{30}h = 1-h$

$\frac{9}{10}h + h = 1$

$\frac{19}{10}h = 1 \longrightarrow h = \frac{10}{19}$

$K = \frac{9}{10} \times \frac{10}{19} = \frac{9}{19}$

(c) Determine the ratio OT:TY (1 mark)

OT : TY

$= \frac{9}{19} : 1 - \frac{9}{19} = \frac{9}{19} : \frac{10}{19}$

$\Rightarrow$ OT : TY = 9 : 10

22. If $\left(x - 1\frac{1}{8}\right)$, x and $\left(x + \frac{3}{2}\right)$ are the first three consecutive terms of a geometric progression;

(a) Determine the values of x and the common ratio. (4 marks)

$$\Rightarrow \frac{x}{x - \frac{9}{8}} = \frac{x + \frac{3}{2}}{x}$$

$$\Rightarrow \frac{x}{\frac{8x-9}{8}} = \frac{\frac{2x+3}{2}}{x}$$

$$\Rightarrow \frac{8x}{8x-9} = \frac{2x+3}{2x}$$

$$\Rightarrow 8x + 2x = (2x+3)(8x-9)$$

$$16x^2 = 2x(8x-9) + 3(8x-9)$$

$$\cancel{16x^2} = \cancel{16x^2} - 18x + 24x - 27$$

$$0 = 6x - 27$$

$$\frac{27}{6} = \frac{6x}{6} \Rightarrow x = \frac{9}{2} \text{ or } 4.5$$

$$r = \frac{9}{2} \div \left(\frac{9}{2} - \frac{9}{8}\right) = \frac{9}{2} \div \frac{27}{8}$$

$$= \frac{9}{2} \times \frac{8}{27} = \frac{4}{3}$$

(b) Calculate the sum of the first 6 terms of this progression. (3 marks)

$$\text{Sum} = \frac{a(r^n - 1)}{r - 1}$$

$$a = \frac{9}{2} - \frac{9}{8} = \frac{27}{8} \text{ or } 3.375$$

$$r = \frac{4}{3},\ n = 6$$

$$\text{Sum} = \frac{\frac{27}{8}\left(\left(\frac{4}{3}\right)^6 - 1\right)}{\frac{4}{3} - 1}$$

$$\text{Sum} = \frac{\frac{27}{8}\left(\frac{3367}{729}\right)}{\frac{1}{3}}$$

$$\text{Sum} = \frac{3367}{216} \times \frac{3}{1}$$

$$\text{Sum} = \frac{3367}{72}$$

$$\text{Sum} = 46.76388$$

(c) Another sequence has the terms

-13, -16, -19, ...-310.

Find the sum of this sequence. (3 marks)

$$\text{Sum} = \frac{n}{2}(a + L)$$

$$= \frac{100}{2}(-13 + -310)$$

$$= 50(-323)$$

$$\text{Sum} = -16,150$$

$$d = -16 + 13 = -3$$

$$L = -310$$

$$n^{th} = a + (n-1)d$$

$$-310 = -13(n-1) - 3$$

$$-310 = -13 + (-3n + 3)$$

$$-310 = -13 - 3n + 3$$

$$-310 = -10 - 3n$$

$$-310 + 10 = -3n$$

$$\frac{-300}{-3} = \frac{-3n}{-3}$$

$$n = 100$$