SECTION I (50 Marks)

Answer all the questions in the spaces provided in this section.

1. Use logarithm tables correct to 4 significant figures to evaluate. (3 Marks)

$$\frac{\sqrt[3]{0.07214 \times 2.037}}{69.8}$$

No	std	Log	Log
0.07214	7.214×10^{-2}	$\bar{2}.8582$	
2.037	2.037×10^{0}	0.3090^{+}	
		$\bar{1}.1672$	
		$\times \frac{1}{3}$	
		$= \frac{-3}{3} + \frac{2.1672}{3}$	
		$= \bar{1}.7224$	

No	std	Log	Log
69.8	6.98×10^{1}		$\bar{1}.7224$
			1.8039^{-}
	7.510×10^{-3}		$\bar{3}.8785$
			$= 0.00756$

2. Solve for x in the equation (2 marks)

$$log\,(5x - 15) - log(2x - 3) = 1$$

$\Rightarrow \log(5x-15) - \log(2x-3) = \log 10$

$\Rightarrow \log \frac{5x-15}{2x-3} = \log 10$

$\Rightarrow \frac{5x-15}{2x-3} = \frac{10}{1}$

$\Rightarrow 5x - 15 = 10(2x-3)$

$5x - 15 = 20x - 30$

$5x - 20x = -30 + 15$

$\frac{-15x}{-15} = \frac{-15}{-15}$

$x = 1$

3. Given that z varies directly as the square of x and inversely as the square root of y. If $x = 2, y = 9$ when $z = 3$, find z when $x = 3$ and $y = 4$. (3 marks)

$z \propto \frac{x^2}{\sqrt{y}}$

$\Rightarrow z = \frac{kx^2}{\sqrt{y}}$

$\Rightarrow 3 = \frac{2^2 k}{\sqrt{9}}$

$3 = \frac{4k}{3}$

$\frac{4k}{4} = \frac{9}{4}$

$k = \frac{9}{4}$

$\Rightarrow z = \frac{\frac{9}{4}x^2}{\sqrt{y}}$

when $x = 3$ and $y = 4$

$z = \frac{\frac{9}{4} \times 3^2}{\sqrt{4}} = \frac{\frac{9}{4} \times 9}{2} = \frac{81}{4} \times \frac{1}{2}$

$= \frac{81}{8} = 10\frac{1}{8}$

4. Evaluate $\frac{1+\sqrt{5}}{2+\sqrt{5}} + \frac{1-\sqrt{5}}{2-\sqrt{5}}$ (3 marks)

$a^2 - b^2 = (a+b)(a-b)$

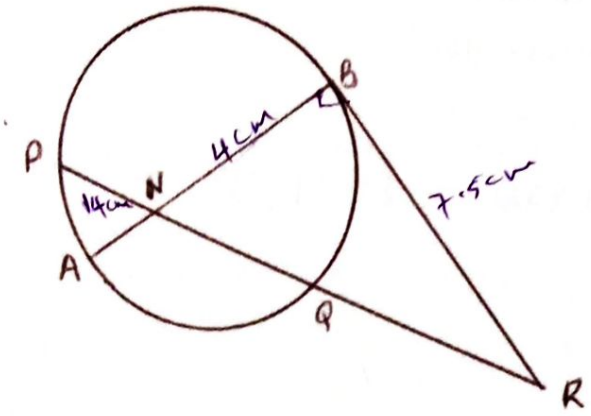
LCM $\frac{(1+\sqrt{5})(2-\sqrt{5}) + (2+\sqrt{5})(1-\sqrt{5})}{(2+\sqrt{5})(2-\sqrt{5})}$

$= \frac{1(2-\sqrt{5}) + \sqrt{5}(2-\sqrt{5}) + 2(1-\sqrt{5}) + \sqrt{5}(1-\sqrt{5})}{4-5}$

$= \frac{2 - \sqrt{5} + 2\sqrt{5} - 5 + 2 - 2\sqrt{5} + \sqrt{5} - 5}{-1}$

$= \frac{-6}{-1} = 6$

5. In the figure below, AB is a diameter of the circle. Chord PQ intersects AB at N. A tangent to the circle at B and meets PQ produced at R.



Given that PN = 14 cm, NB = 4 cm and BR = 7.5 cm, calculate the length of;

a) NR (2 marks)

$NR = \sqrt{7.5^2 + 4^2} = \sqrt{72.25} = 8.5\text{cm}$.

b) AN (2 marks)

$PR = 14 + 8.5 = 22.5$

$PR \times RQ = BR^2$ let $NQ = x \Rightarrow QR = 8.5 - x$

$\Rightarrow 22.5(8.5 - x) = 7.5^2$

$191.25 - 22.5x = 56.25$

$191.25 - 56.25 = 22.5x$

$\frac{135}{22.5} = \frac{22.5}{22.5}x$

$x = \frac{135}{22.5} = 6$

$\Rightarrow AN \times NB = PN \times NQ$

$AN \times 4 = 14 \times 6$

$AN = \frac{14 \times 6}{4} 3 = 21\text{cm}$ ✓

6. Calculate the semi-interquartile range of 3,4,1,2,3,6,8,5,7,9. (3 marks)

Arrange the numbers in Ascending order

1 2 3 3 4 5 6 7 8 9

$Q_1 = \left(\frac{n+1}{4}\right)^{th}$ term $= \frac{10+1}{4} = 2.75$ term $= \frac{2+3}{2} = 2.5$

$Q_3 = \frac{3(n+1)}{4}^{th}$ term $= \frac{3(11)}{4} = 8.25$ term $= \frac{7+8}{2} = 7.5$

Quartile deviation $= \frac{Q_3 - Q_1}{2} = \frac{7.5 - 2.5}{2} = \frac{5}{2} = 2.5$ ✓

7. Given that a = 3i − 2j + 3k and b = 2i − 4j − 3k where i, j and k are unit vectors, find

|2a + 3b| (3 Marks)

$2a = 2(3i-2j+3k) = 6i-4j+6k$

$3b = 3(2i-4j-3k) = 6i-12j-9k$

$\Rightarrow 2a+3b = 12i-16j-3k$

$|2a+3b| = \sqrt{12^2+(-16)^2+(-3)^2} = \sqrt{409} = 20.2237 \text{ units}$

8. Make x the subject of the formula in

$$a = \left(\frac{bx}{b+x}\right)^{\frac{1}{3}}$$

Hence find the value of x when a = 2 and b = 6. (4 Marks)

$\left[a = \left(\frac{bx}{b+x}\right)^{\frac{1}{3}}\right]^3$

$a^3 = \frac{bx}{b+x}$

$\Rightarrow a^3(b+x) = bx$

$a^3b + a^3x = bx$

$a^3b = bx - a^3x$

$a^3b = x(b-a^3)$

$\frac{a^3b}{b-a^3} = \frac{x(b-a^3)}{b-a^3}$

$x = \frac{a^3b}{b-a^3}$ ✓

When $a=2, b=6$

$x = \frac{2^3\times 6}{6-2^3} = \frac{48}{-2} = -24$ ✓

9. Expand $(a-x)^6$ upto the term in $(x)^3$. Use your expansion to estimate the value of $(2.99)^6$ to 3 decimal places. (4 Marks)

coefficients 1, 6, 15, 20, 15, 6, 1

$(a-b)^6 = 1(a^6)(x)^0 - 6(a)^5(x)^1 + 15(a)^4(x)^2 - 20(a)^3(x)^3$

$= a^6 - 6a^5x + 15a^4x^2 - 20a^3x^3$

$\Rightarrow (a-x)^6 = (2.99)^9 \Rightarrow (a-x)^6 = (3-0.01)^9$

$\Rightarrow a = 3, \; x = 0.01$

$\Rightarrow a^6 - 6a^5x + 15a^4x^2 - 20a^3x^3 = 3^3 - 6(3)^5(0.01) + 15(3)^4(0.01)^2 - 20(3)^3(0.01)^3$

$= 729 - 14.58 + 0.1215 - 0.00054$

$= 714.54096 = 714.541$ (3 d.p) ✓

10. A coffee blender has two brands of coffee, Tamu and Chungu. A kilogram of Tamu costs Sh. 70 while a kilogram of Chungu costs Shs. 64. In what ratio should he mix the two brands to make a blend which costs Shs. 68 per kilogram? (2 Marks)

let x kg of Tamu be mixed with y kg of Chungu.

$\Rightarrow 70x + 64y = 68(x+y)$

$\Rightarrow 70x + 64y = 68x + 68y$

$\Rightarrow 70x - 68x = 68y - 64y$

$2x = 4y$

$\frac{2x}{2} = \frac{4y}{2}$

$\frac{1}{y} \times x = \frac{2}{1} y \times \frac{1}{y}$

$\frac{x}{y} = \frac{2}{1} \therefore x:y = 2:1$ ✓

11. Find the centre and radius of a circle whose equation is $x^2 + y^2 + 8x - 2y - 1 = 0$ (3 Marks)

$\Rightarrow x^2 + 8x + y^2 - 2y = 1$

$\Rightarrow x^2 + 8x + 16 + y^2 - 2y + 1 = 1 + 16 + 1$

$(x+4)^2 + (y-1)^2 = 18$ ✓

From $(x-a)^2 + (y-b)^2 = r^2$ where centre $= (a, b)$, $r = \sqrt{r^2}$

$\Rightarrow$ centre $= (-4, 1)$ ✓

radius $= \sqrt{18} = 4.243$ units. ✓

12. The original area of an object after two successive transformations given by $\begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}$ and $\begin{bmatrix} 3 & 1 \\ 0 & 1 \end{bmatrix}$ in that order becomes 168 square units. Find the original area of the object. (3 marks)

Single matrix $\begin{pmatrix} 3 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 3 & 5 \end{pmatrix} = \begin{pmatrix} 9 & 8 \\ 3 & 5 \end{pmatrix}$ ✓

$\det = (9 \times 5) - (8 \times 3) = 21$ ✓

$\frac{\text{Area of image}}{\text{Area of object}} = \det$

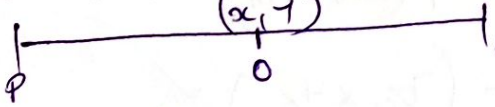
$\frac{168}{x} = \frac{21}{1}$

$21x = 168$

$x = \frac{168}{21} = 8$ sq units. ✓

13. Find the equation of the perpendicular bisector of line PQ where the co-ordinates of P and Q are P (-2, 8) and Q (4, 7). (3 Marks)

distance $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$



Using distance formula

$\Rightarrow |PO| = |OP|$

$\Rightarrow (x+2)^2 + (y-8)^2 = (x-4)^2 + (y-7)^2$ ✓

$\Rightarrow x^2 + 4x + 4 + y^2 - 16y + 64 = x^2 - 8x + 16 + y^2 - 14y + 49$

$\Rightarrow 4x + 8x + 4 + 64 - 16 - 49 = -14y + 16y$

$12x + 3 = 2y$

$\therefore 2y = 12x + 3$

or $y = 6x + \frac{3}{2}$ ✓

14. The surface areas of two spheres are $36cm^2$ and $49cm^2$. If the volume of the smaller sphere is $20.2cm^3$, calculate the volume of the larger one. (2 Marks)

Area scale factor = $\frac{\text{Area of image}}{\text{Area of object}}$

$\Rightarrow$ A.s.f = $\frac{49}{36}$

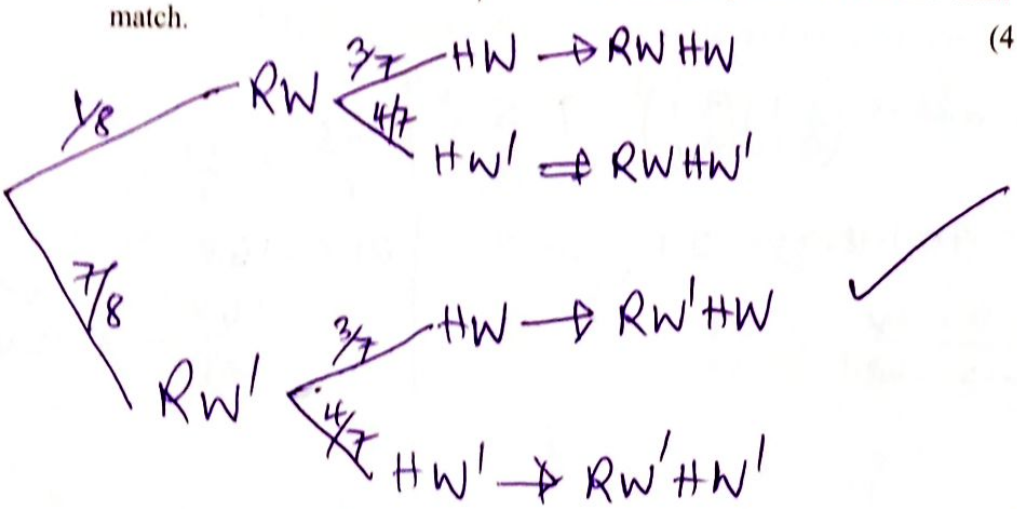
$\Rightarrow$ L.s.f = $\sqrt{\frac{49}{36}} = \frac{7}{6}$

V.s.f = $(\text{L.s.f})^3 = (\frac{7}{6})^3 = \frac{343}{216}$

$\Rightarrow \frac{x}{20.2} = \frac{343}{216}$

$x = \frac{343 \times 20.2}{216} = \frac{6928.6}{216} = 32.0769cm^3$ ✓

15. During inter-school competitions, rugby and football teams from Ranje sec school took part. The probability that the rugby would win their first match was $\frac{1}{8}$ while that the handball team could lose was $\frac{4}{7}$. Find the probability that at least one team won the first match. (4 marks)



P(atleast 1 team won) = 1 − P(RW' HW') ✓

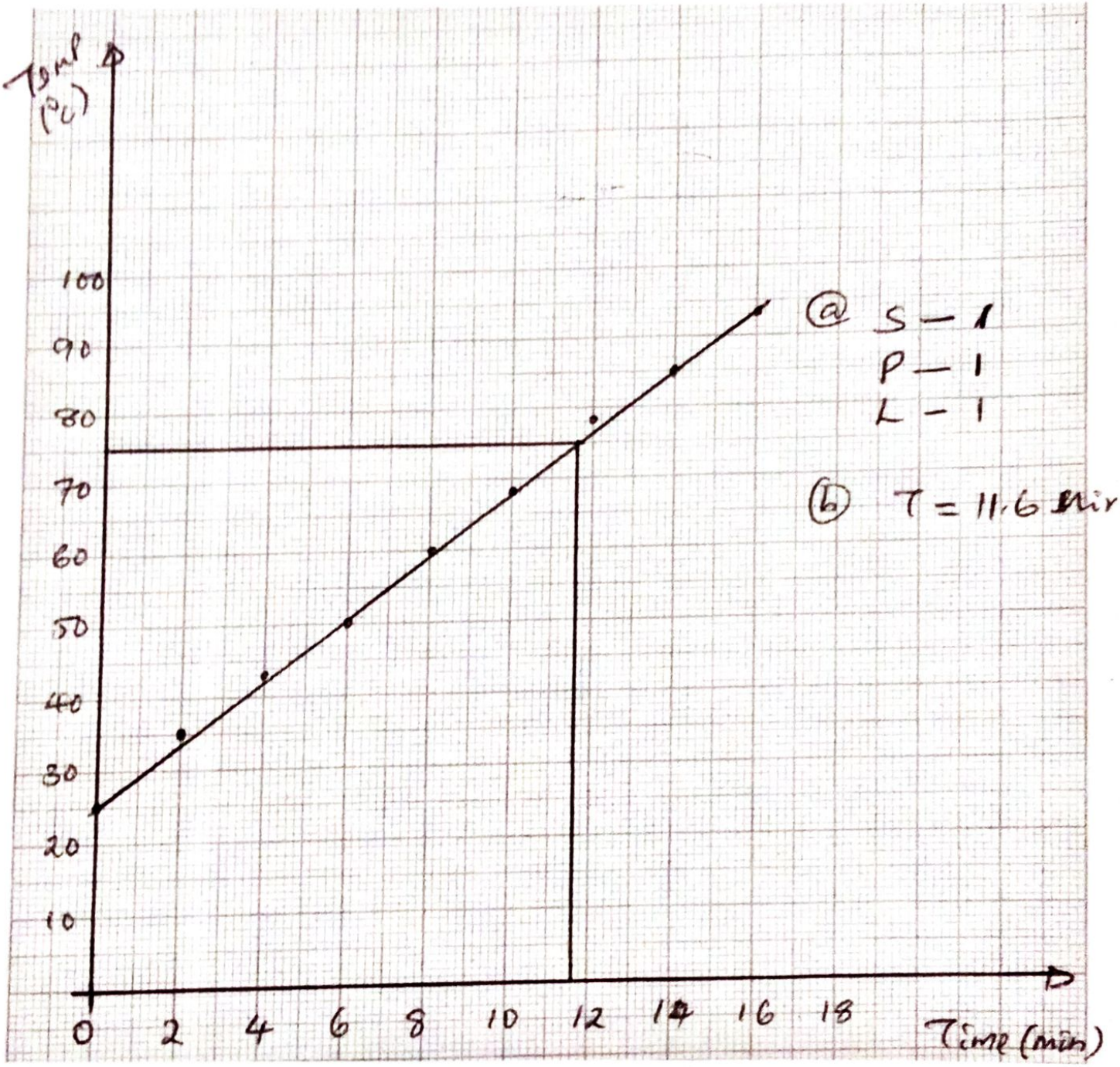
= 1 − (7/8 × 4/7) ✓

= 1 − 1/2 = 1/2 ✓

16. In an experiment, water was heated and its temperature changes recorded at intervals of 2 minutes as shown in the table below.

Time (Min)	0	2	4	6	8	10	12	14	16
Temperature (°C)	25	35	42.5	50	60	67.5	77.5	85	92.5

a) On the grid provided, plot the points and draw the line of best fit. (3 marks)



b) Use the line of best fit to estimate the time taken for the temperature of the water to be 75°C. (1 mark)

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