SECTION I (50 MARKS)

Answer all the questions from this section

1. Without using a calculator evaluate (3 Marks)

$$\frac{\left(3\frac{1}{3}+1\frac{1}{9}\right)\div 1\frac{1}{3}}{\left(4\frac{2}{9}-2\frac{5}{9}\right)\times\frac{2}{3}}$$

$= \frac{\left(10/3 + 10/9\right) \div 4/3}{\left(38/9 - 23/9\right) \times 2/3}$

$= \frac{30+10}{9} \times \frac{3}{4} \div \frac{15}{9} \times \frac{2}{3}$

$= \frac{40}{9} \times \frac{3}{4} \div \frac{15}{9} \times \frac{4}{3}$

$= \frac{10}{3} \times \frac{9}{10} = 3$

2. (a) Use mathematical tables to find the:

(i) The square of 86.46 (1 mark)

$(8.646 \times 10^1)^2$

$8.646^2 \times 10^2$

$= 74.65 + 0.10$

$= 74.75 \times 100 = 7475$

(ii) The reciprocal of 27.56 (1 mark)

$\frac{1}{27.56} = 2.756 \times 10^{-1} = 0.3628 \times \frac{1}{10}$

$= 0.036\,28.$

(b) Hence or otherwise calculate the value of; (2 marks)

$$\frac{86.46^2}{27.56}$$

$= 7475 \times 0.036\,28$

$= 271.192$

$= 271.2$ (1dp)

3. The sum of the interior angles of an n – sided polygon is 1440^0. Find the value of n and hence deduce the name of the polygon. (3 marks)

Sum $= (2n - 4)90$

$= 180n - 360$

$1440 = 180n - 360$

$1440 + 360 = 180n$

$\frac{1800}{180} = \frac{180n}{180}$

$n = 0$

$\therefore$ Decagon.

4. Solve for x and y in the equation. (3 Marks)

$2^{(5y+1)} \times 5^{(2x+2)} = 1250$

$2^{(5y+1)} \times 5^{(2x+2)} = 5^4 \times 2^1$

$2^{(5y+1)} = 2^1$

$5y + 1 = 1$

$5y = 0$

$y = 0.$

$5^{(2x+2)} = 5^4.$

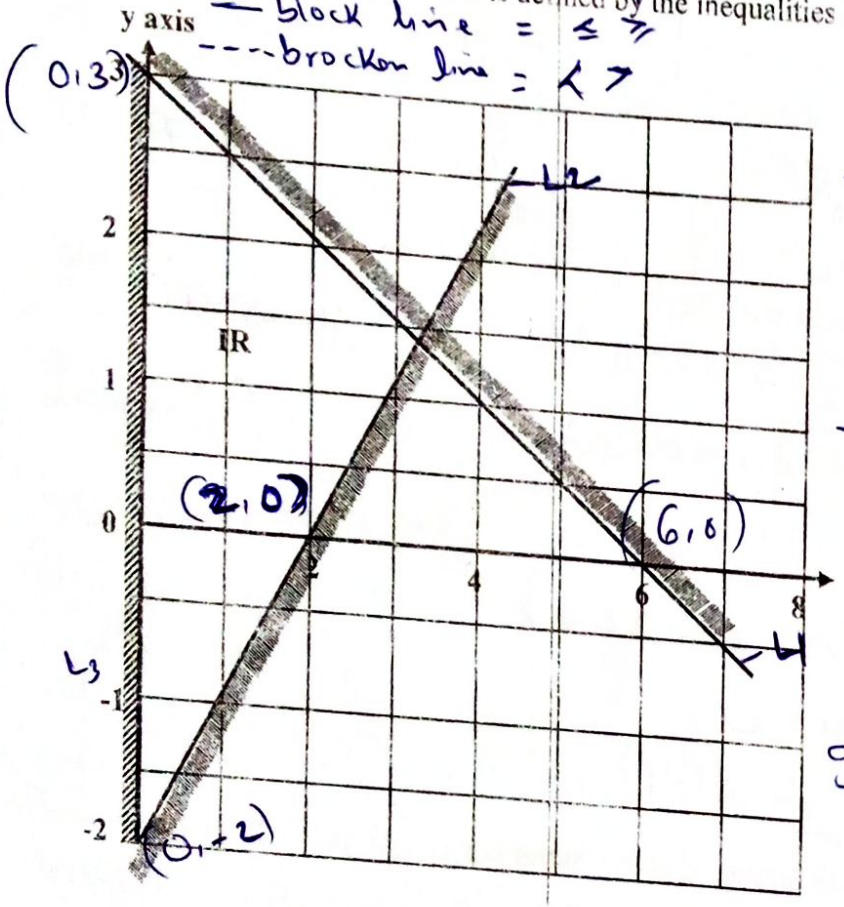
$2x + 2 = 4.$

$2x = 2$

$x = 1$

$x = 1, \ y = 0$

5. The region R in the figure below is defined by the inequalities L1, L2 and L3.



— block line = $\leq$ or $\geq$

---- broken line = $<$ or $>$

Find the three inequalities (3 Marks)

$L_3 \Rightarrow x \geq 0$

L_1

gradient $= \frac{0-3}{6-0} = -\frac{3}{6} = -\frac{1}{2}$

$-\frac{1}{2} = \frac{y+0}{x-6} \Rightarrow 2y = -x + 6$

$y = -\frac{1}{2}x + 3$

Test point $(1,2)$

$2 = 2.5 \Rightarrow 2 < 2.5$

$\therefore y \leq -\frac{1}{2}x + 3.$

L_2

gradient $= \frac{0+2}{2-0} = \frac{2}{2} = 1$

$\frac{1}{1} = \frac{y-0}{x-2}$

$y = x - 2$

Test point $= (1,2)$

$2 = 1 - 2 \Rightarrow 2 = -1$

$2 > -1$

$\therefore y = x - 2$

6. Two boys and a girl shared some money. The elder boy got $\frac{4}{9}$ of it, the younger boy got $\frac{2}{5}$ of the remainder and the girl got the rest. Find the percentage share of the younger boy to the girl's share (3Marks)

Let the amount be x.

elder boy $= 4/9\ x$

younger boy $= 2/5 \times 5/9 = 2/9\ x$

girl $= x - (4/9\ x + 2/9\ x) = x - 6/9\ x = 3/9\ x$

$= 1/3\ x$

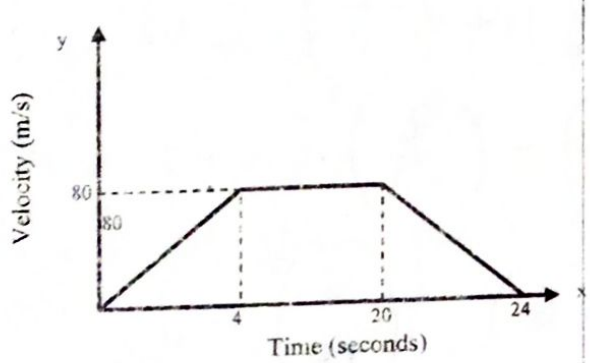
younger boy : girl

$2/9\ x : 1/3\ x$.

$\therefore \frac{2/9\ x}{1/3\ x} \times 100$

$= 66.67\%$

7. The figure below is a velocity – time graph for a car. (not drawn to scale).



(a) Find the total distance traveled by the car? (2 Mks)

Distance = Area of trapezium

$= \frac{1}{2}(24 + 16) \times 80$

$= \frac{1}{2} \times 40 \times 80 = 1600M$.

(b) Calculate the deceleration of the car. (2 Marks)

Deceleration $= \frac{v - u}{t}$

$u = 80$ m/s

$v = 0$ m/s.

$t = 4$ s.

$= \frac{0 - 80}{4} = -80/4 = 20 m/s^2$.

8. Two containers have base areas of 750cm² and 120cm² respectively. Calculate the volume of the larger container in litres given that the volume of the smaller container is 400cm³. (3 marks)

As. F $= \frac{750}{120} = 25/4$

(A.F)

L.s.F $= 5/2$

V.s.F $= (\text{L.s.f})^3 = 5^3/2^3 = 125/8$

Let Volume be V.

$\frac{125}{8} = \frac{V}{400}$

$V = \frac{400 \times 125}{8}$

$V = 6250\ cm^3$

in litres $= \frac{6250}{1000} = 6.25l$

9. Given that the column vectors $\boldsymbol{a} = \begin{pmatrix} -1 \\ 4 \end{pmatrix}, \boldsymbol{b} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$ and $\boldsymbol{c} = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$ and that $\boldsymbol{P} = 2a - 4b + 3c$. Express P as a column vector. (3 marks)

$P = 2\begin{pmatrix} -1 \\ 4 \end{pmatrix} - 4\begin{pmatrix} -3 \\ -2 \end{pmatrix} + 3\begin{pmatrix} -2 \\ -1 \end{pmatrix}$

$= \begin{pmatrix} -2 \\ 8 \end{pmatrix} - \begin{pmatrix} -12 \\ -8 \end{pmatrix} + \begin{pmatrix} -6 \\ -3 \end{pmatrix}$

$= \begin{pmatrix} -2 + 12 - 6 \\ 8 + 8 - 3 \end{pmatrix} = \begin{pmatrix} 4 \\ 13 \end{pmatrix}$

10. Simplify the following expression by reducing it to a single fraction. (3 marks)

$\frac{2x-3}{3} - \frac{x-2}{2} - \frac{1-x}{4}$

$= \frac{4(2x-3) - 6(x-2) - 3(1-x)}{12}$

$= \frac{8x - 12 - 6x + 12 - 3 + 3x}{12}$

$= \frac{5x - 3}{12}$

11. Thirty men working at a rate of 10 hours a day can complete a job in 14 days. Find how long it would take 40 men working at the rate of 7 hours a day to complete the same job.(3 marks)

M	H	D
30	10	14
40	7	x

$$x = \frac{30}{40} \times \frac{10}{7} \times 14 = 15 \text{ days.}$$

12. A car uses 1 litre of petrol for every 8 kilometres. The car was to travel 480 kilometres and had 15 litre of petrol at the beginning of the journey. Each litre of petrol cost sh. 112.00. How much did it cost for the extra petrol added? (3mks)

Total litres used $= \frac{480}{8} = 60$ litres

Extra petrol $= 60 - 15 = 45$ litres.

Cost of Extra petrol $= 45 \times 112$

$= \text{Ksh } 5,040.$

13. (a) Find the greatest common divisor of the terms $9x^3y^2$ and $4xy^4$ (1 mark)

G.C.D $= xy^2$

(b) Hence factorize completely the expression (2 marks)

$9x^3y^2 - 4xy^4$

$= xy^2(9x^2 - 4y^2)$

$= xy^2(3x - 2y)(3x + 2y)$

14. A rectangular block is 50cm long and 15 cm wide. If its mass is 18kg and its density is 2.4g/cm³, find its height. (3 marks)

$D = M/V$

$V = M/D$.

$2.4 = \frac{18 \times 1000}{15 \times 50 \times h}$

$750h = \frac{18000}{2.4}$

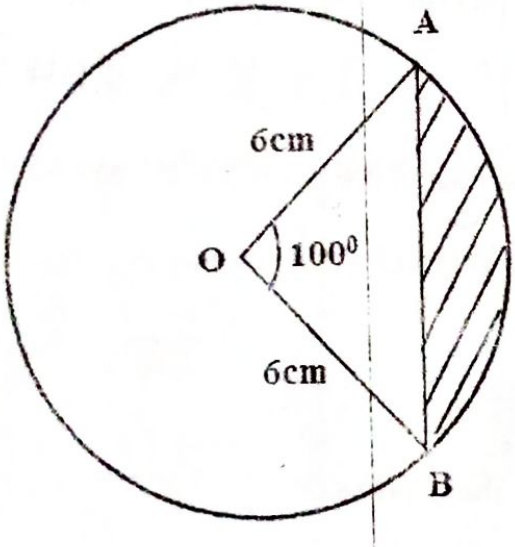
$h = 1\text{ cm}$.

15. Use the prime factors of 1764 and 2744 to evaluate (3 marks)

$\frac{\sqrt{1764}}{\sqrt[3]{2744}} = \frac{\sqrt{2^2 \times 3^2 \times 7^2}}{\sqrt[3]{2^3 \times 7^3}}$

$= \frac{2 \times 3 \times 7}{2 \times 7} = 3//$

16. The figure below shows a circle centre O and radius 6cm. sector OAB subtends an angle of 100^0 at the centre of the circle as shown.



Calculate to 2 decimal places the area of the shaded region. (Take $\pi = \frac{22}{7}$) (3 marks)

Area of Segment $= \frac{r^2}{2}\left(\frac{\theta\pi}{180} - \sin\theta\right)$

$= \frac{36}{2}\left(\frac{100 \times 22/7}{180} - \sin 100^\circ\right)$

$= 18(1.746031 - 0.9848)$

$= 18 \times 0.761231$

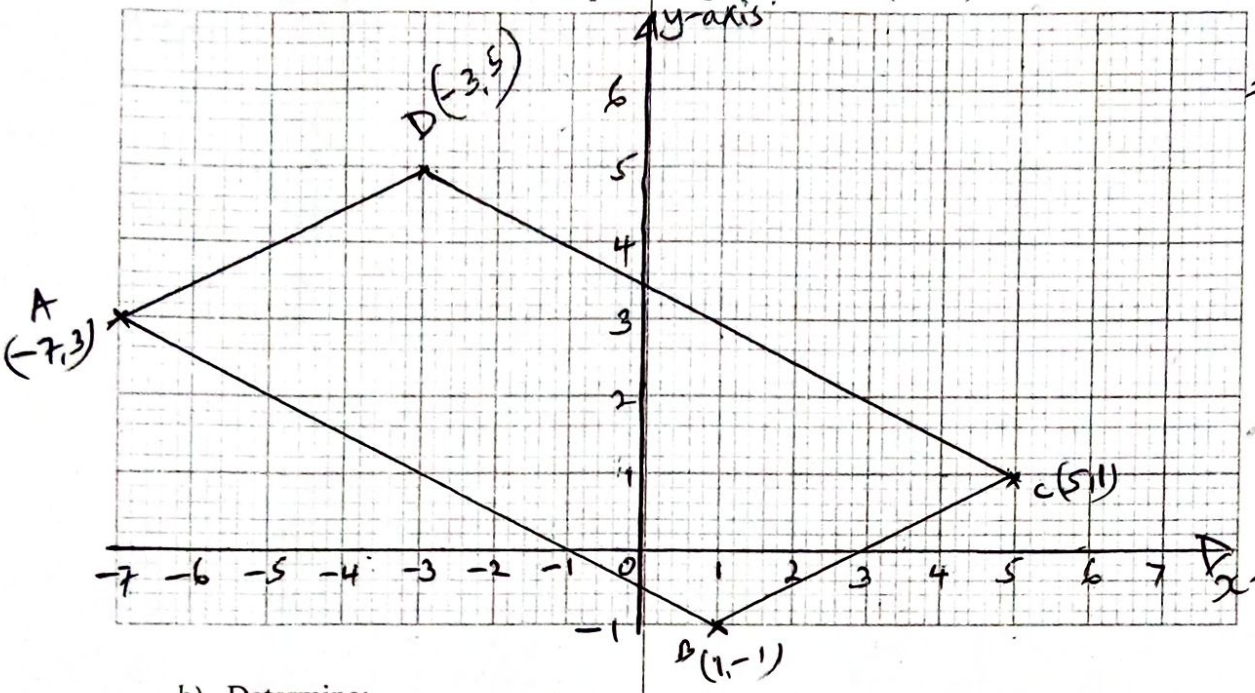
$= 13.702158 = 13.70\text{ cm}^2$

SECTION II (50 MARKS)

Answer FIVE questions ONLY from this section

17.

a) Three vertices of a parallelogram ABCD are A(−7,3), B(1,−1) and C(5,1). On the grid provided, draw the parallelogram ABCD. (2mks)



Identify D

Gradient of AD = DC

$\Rightarrow \frac{3+1}{-7-1} = \frac{y-1}{x-5}$

$y-1=4$ and $x-5=-8$

$y=5$ $x=-3$

$\therefore D(-3,5)$ ✓

b) Determine:

(i) the gradient of line AB.(2mks)

Gradient of AB $= \frac{\Delta y}{\Delta x} = \frac{3+1}{-7-1} = \frac{4}{-8} = -\frac{1}{2}$ ✓

(ii) the equation of line AB in the form $y = mx + c$, where m and are constants.(2mks)

$\Rightarrow$ Let another point (x,y) be on AB.

$\Rightarrow -\frac{1}{2} = \frac{y-3}{x-7} \Rightarrow -\frac{1}{2} = \frac{y-3}{x+7} \Rightarrow 2y-6 = -x-7$

$2y = -x-7+6$

$2y = -x-1$

$y = -\frac{1}{2}x - \frac{1}{2}$ ✓

c) Another line L is perpendicular to CD and passes through the point (1, 3). Determine:

(i) the equation of L in the form $ax + by = c$ where a, b and c are constants (3 marks)

Gradient of CD = gradient of AB $= -\frac{1}{2}$.

$\Rightarrow$ let gradient of L be M

$\Rightarrow -\frac{1}{2} \times M = -1$

$\Rightarrow M = 2$

another point (x,y) be on L

$\Rightarrow \frac{2}{1} = \frac{y-3}{x-1}$

$\Rightarrow y-3 = 2x-2$

$y = 2x+1$

$\Rightarrow 2x - y = -1$ (form $ax+by=c$)

(ii) the coordinates of the y-intercept of line L. (1 mark)

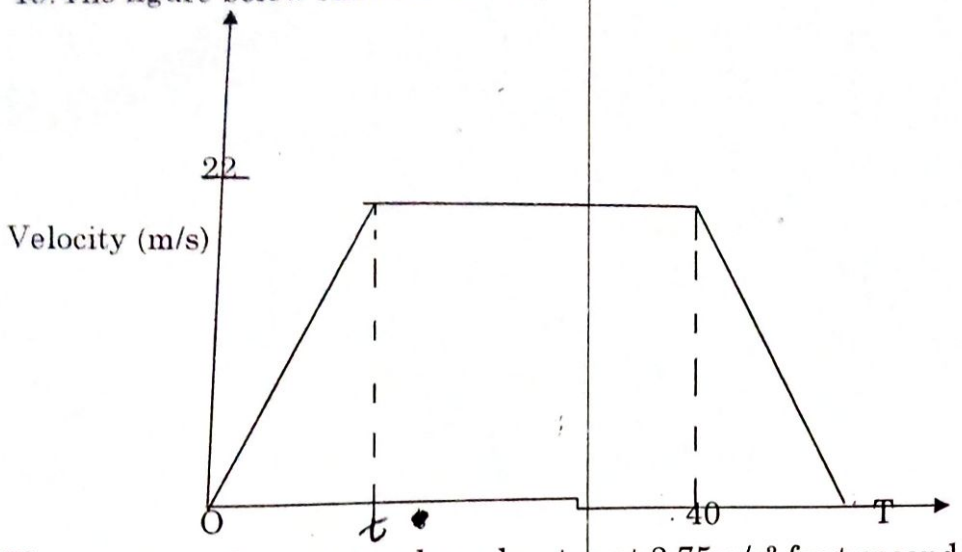
at y intercept $x=0$

$\Rightarrow 2x - y = -1$ sub $x=0$

$\Rightarrow 0 - y = -1$

$\Rightarrow y = 1$ ✓ cordinate $(0,1)$

19. The figure below shows a velocity – time graph of a car journey.



The car starts from rest and accelerates at 2.75m/s² for t seconds until its speed is 22m/s. It then travels at this velocity until 40 seconds after starting. Its breaks bring it uniformly to rest. The total journey is 847m long and takes T seconds.
Calculate the

(i) Value of t (3mks)

acceleration = $\frac{\text{Final velocity} - \text{initial velocity}}{\text{Time taken}}$

$\Rightarrow 2.75\text{m/s}^2 = \frac{22\text{m/s} - 0\text{m/s}}{t}$ ✓(1)

$t = \frac{22}{2.75}$ ✓(1)

$t = 8$ seconds. ✓(1)

(ii) Distance travelled during the first t seconds (2mks)

distance = Area under the curve;

$= \frac{1}{2} \times 8 \times 22 = 88\text{m}$ ✓

(iii) Value of T (3mks)

Area under the curve = total distance

$\Rightarrow 847 = \frac{1}{2} \times 22 (T + 32)$ ✓

$847 = 11T + 352$

$495 = 11T$ ✓

$T = \frac{495}{11} = 45$ seconds. ✓

(iv) Final deceleration (2mks

$\frac{0 - 22}{45 - 40} = \frac{-22}{5} = 4.4\text{m/s}^2$ ✓

deceleration $= 4.4\text{m/s}^2$

or $a = -4.4\text{ms}^{-2}$.

20. A school in Meru Central decided to buy x calculators for its students for a total cost of ksh. 16,200. The supplier agreed to offer a discount of ksh. 60 per calculator. The school was then able to get three extra calculators for the same amount of money.

(a) Write an expression in terms of x , for the

(i) Original price of each calculator (1mk)

$\frac{16,200}{x}$ ✓ (1)

(ii) Price of each calculator after the discount (1mk)

$\frac{16\,200}{x+3}$ ✓ (1)

a) Form an equation in x and hence determine the number of calculators the school bought (5mks)

$\Rightarrow \frac{16\,200}{x} - \frac{60}{1} = \frac{16200}{x+3}$

$\frac{16200-60x}{x} = \frac{16200}{x+3}$

$\Rightarrow (x+3)(16200-60x) = 16200x$

$x(16200-60x)+3(16200-60x) = 16200x$

$16200x - 60x^2 + 48600 - 180x = 16200x$

$\Rightarrow -60x^2 - 180x + 48600 = 0$

$60x^2 + 180x - 48600 = 0$

$x^2 + 3x - 810 = 0$

$x = \frac{-3 \pm \sqrt{9+3240}}{2}$

$x = \frac{-3 \pm 57}{2}$

$x = \frac{54}{2}$ or $\frac{-60}{2}$

$x = 27$ or -30

∴ No of calculators were 27 ✓ II

b) Calculate the discount offered to the school as a percentage (3mks)

Initial cost $= \frac{16200}{27} = sh\ 600$ ✓ (1)

discount $= sh\ 60$.

% discount $= \frac{60}{600} \times 100$ ✓ (1)

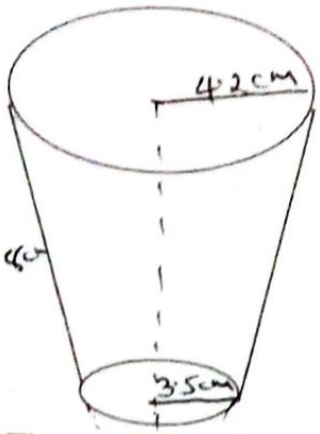
$= 10\%$ ✓ (1)

6

21. A solid is made up of a conical frustum and a hemispherical top. The slant height of the frustum is 8cm and its base radius is 3.5cm. If the radius of the hemispherical top is 4.2cm.

(a) Find the area of:

(i) The circular base. (2 Marks)



Area of circular base $= \pi r^2$

$= \frac{22}{7} \times 3.5 \times 3.5$

$= 38.5\ cm^2$

(ii) The curved surface of the frustum (3 Marks)

$\frac{4.2}{3.5} = \frac{8+x}{x}$

$\Rightarrow 4.2x = 28 + 3.5x$

$0.7x = 28$

$x = 40cm$

Curved surface area $= \pi RL - \pi rl$

$= \left(\frac{22}{7} \times 42 \times 48\right) - \frac{22}{7} \times 3.5 \times 40$

$= 633.6 - 440 = 193.6\ cm^2$

(iii) The hemispherical surface (3 Marks)

Area $= 2\pi r^2$

$= 2 \times \frac{22}{7} \times 4.2 \times 4.2$

$A = 776.16\ cm^2$

(b) A similar solid has a total surface area of 81.51cm². Determine the radius of its base. (2 Marks)

Total surface area of solid $= 38.5cm^2 + 193.6cm^2 + 776.16cm^2$

$= 1008.26cm^2$

$A.S.f = \frac{1008.26}{81.51} = \frac{12.36977}{1}$

$L.S.f = \sqrt{12.36977} = \frac{3.517}{1}$

$\frac{3.517}{1} = \frac{3.5}{r}$

$r = 3.5 = 0.995\ cm$

22. The table below shows the amount in shillings of pocket money given to students in a particular school.

Pocket money (Kshs)	201 – 219	220 – 229	230 – 239	240 – 249	250 – 259	260 – 269	270 – 279	280 – 289	290 – 299
No. of students	5	13	23	32	26	20	15	12	4

(a) State the modal class. (1 mk)

240 – 249

(b) Calculate the mean amount of pocket money given to these students to the nearest shilling. (4 mks)

Class	X	F	FX	C.F
201-219	210	5	1050	5
220-229	224.5	13	2918.5	18
230-239	234.5	23	5393.5	41
240-249	244.5	32	7824	73
250-259	254.5	26	6617	99
260-269	264.5	20	5290	119
270-279	274.5	15	4117.5	134
280-289	284.5	12	3414	146
290-299	294.5	4	1178	150

$\Sigma F = 150$

$\Sigma Fx = 37802.5$

$\text{Mean} = \frac{\Sigma Fx}{\Sigma F} = \frac{37802.5}{150}$

$= 252.017$

(c) Calculate the median amount of money given to a student. (5 mks)

Median class $= \frac{1}{2} \times 150 = 75$

$= 250 - 259$

$\text{Median} = L_1 + \left(\frac{\frac{N}{2} - ca}{f_m}\right) i$

$= 249.5 + \left(\frac{75 - 73}{26}\right) \times 10$

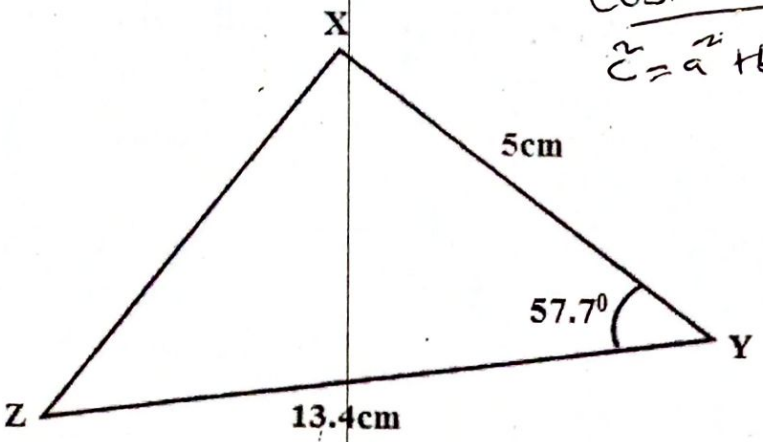
$= 249.5 + \frac{2}{26} \times 10$

$= 249.5 + 0.769230$

$= 250.26923$

23. The figure below shows triangle XYZ in which line XY = 5cm, line YZ = 13.4cm and the size of angle XYZ=57.7^0

Cosine rule:
$c^2 = a^2 + b^2 - 2ab\cos C$



(a) Calculate the length of line XZ (4 marks)

$XZ^2 = 13.4^2 + 5^2 - 2\times13.4\times5\cos 57.7$ ✓

$XZ^2 = 179.56 + 25 - 134\cos57.7$ ✓

$XZ^2 = 204.56 - 71.603$

$XZ^2 = 132.957$ ✓

$XZ = \sqrt{132.957}$

$XZ = 11.53$ cm ✓✓

(b) Calculate the size of angle XZY (4 marks)

Sin Rule $\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{XZ}{\sin 57.7} = \frac{5}{\sin XZY}$

$\frac{11.53}{\sin 57.7} = \frac{5}{\sin XZY}$

$11.53\sin XZY = 5\sin 57.7$

$\sin XZY = \frac{5\sin 57.7}{11.3} = 0.3665$

$XZY = \sin^{-1} 0.3665$ ✓

$\angle XZY = 21.499 \approx 21.5^\circ$ ✓✓

(c) Calculate the size of angle YXZ to 4 significant figures (2 marks)

$\angle YXZ = 180 - [57.7 + 21.5]$ ✓✓

$= 180^\circ - 79.2 = 100.8^\circ$ ✓✓

24. The equation of a curve is $y = 2x^3 - 9x^2 + 12x - 9$ .find:

a) The gradient of the curve when x = 3. (3 marks)

$\frac{dy}{dx} = 6x^2 - 18x + 12$ ✓

When $x = 3$

$\frac{dy}{dx} = 6(3)^3 - (18 \times 3) + 12$ ✓

$\frac{dy}{dx} = 54 - 54 + 12 = 12$ ✓

b) i) The turning points of the curve. (3 marks)

at turning point $\frac{dy}{dx} = 0$

$\Rightarrow 6x^2 - 18x + 12 = 0$ ✓

$x = \frac{18 \pm \sqrt{324 - 288}}{12}$

$x = \frac{18 \pm 6}{12}$

$x = \frac{24}{12} = 2$ or $\frac{12}{12} = 1$ ✓

When $x = 2$

$y = 2(2)^3 - 9(2)^2 + 12(2) - 9 = -5$

When $x = 1$

$y = 2(1)^3 - 9(1)^2 + 12(1) - 9 = -4$

∴ turning points are:

$(1, -4)$ and $(2, -5)$ ✓

ii) The nature of the turning point b(i) above. (2 marks)

$\frac{d^2y}{dx} = 12x - 18$

When $x = 1$

$\frac{d^2y}{dx} = -6$ which is < 0

hence $(1, -4)$ is Max point ✓

When $x = 2$

$\frac{d^2y}{dx} = 6$ which is > 0 ✓

hence $(2, -5)$ is min point ✓

c) Sketch the curve. (2 marks)