NAME ……………………………………………..ADM…………..CLASS …………DATE………….

GURUS PAPER 1
MARKING SCHEME

MATHEMATICS
2 ½ Hours

Instructions to Candidates:

a) *Write your name adm no, class and the date in the spaces provided above.*
b) *This paper contains **two** sections: **Section I** and **Section II.***
c) *Answer **all** questions in **section I** and only **five** questions in **section II.***
d) ***Show all the steps in your calculations, giving your answer at each stage in the spaces below each question.***
e) *Marks may be given for correct working even if the answer is wrong.*
f) ***Non-programmable** silent electronic calculators **and** KNEC Mathematical table may be used, except where stated otherwise.*
g) ***This paper consists of 14 printed pages***
h) ***Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.***
i) ***Candidates should answer the questions in English***

For Examiners use only.
SECTION II

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	**TOTAL**

SECTION II

17	18	19	20	21	22	23	24	**TOTAL**

GRAND TOTAL	

SECTION 1 (50 MARKS)

Answer **all** the questions in this section in the spaces provided.

1. Evaluate without using a calculator (3 marks)

$$\frac{45-8\times{}^{-}4-15\div{}^{-}3}{3\times{}^{-}3+{}^{-}8(6-2)}$$

$$\frac{45\ -\ -32\ -\ -5}{-9\pm8(4)} = \frac{45\ +\ 32\ +\ 5}{-9\ -\ 32} = \frac{82}{-41}$$

M1 (For correct numerator)
M1 (For correct denominator)

$$= -2$$

A1

2. Solve for n in $\frac{y^7 \times y^{11}}{y^4 \times y^n} = y^5$ (3 marks)

Applying the law of multiplication on the numerator and denominator

$$\frac{y^{7\ +\ 11}}{y^{4\ +\ n}} = y^5$$

$$\frac{y^{18}}{y^{4\ +\ n}} = y^5$$

M1 (Applying addition law on both numerator and denominator)

Applying the law of division on the LHS

$$y^{18\ -\ (4\ +\ n)} = y^5$$

$$y^{14\ -\ n} = y^5$$

M1 (Applying the law of division correctly)

Since the bases are equal and the expressions are equal then the powers are equal

$$14 - n = 5$$

$$14 - 5 = n$$

$$9 = n$$

A1

3. A furniture dealer imported 25 Italian-made sofas at Ksh 120 000 each. He sold 10 of them at a profit of 30% and the rest at a discount of Ksh 20 000 each. Calculate his overall profit. (3 marks)

$$\text{Cost Price} = 120000 \times 25$$
$$= Ksh\ 3000000$$

M1 (Cost price)

$$\text{Sales of 10 sofas} = \frac{130}{100} \times 120000 \times 10$$

$$= Ksh\ 1\ 560\ 000$$

$$\text{Sales of 15 sofas} = (120000 - 20000) \times 15$$
$$= \text{Ksh } 1500\ 000$$

$$\text{Total sales} = 1\ 560\ 000 + 1\ 500\ 000$$
$$= \text{Ksh } 3\ 060\ 000$$

M1 (Total sales)

$$\text{Profit} = 3\ 060\ 000 - 3\ 000\ 000$$
$$= \text{Ksh } 60\ 000$$

A1

4. Denis sold 300 tickets for a music concert. He sold adult tickets at sh 500 each and children tickets at sh 400. He collected a total of sh 144 400 in ticket sales. Determine the number children tickets he sold. (3 marks)

Let x be adult and y be children tickets sold then:

$$x + y = 300$$
$$500x + 400x = 144400$$

M1(Forming a pair of simultaneous equations)

$$500x + 500y = 150000$$
$$500x + 400y = 144400 -$$
$$100y = 5600$$
$$y = 56$$

M1 (Correct attempt to solve the pair of equations)

A1

ALTERNATIVELY

Let x be children tickets sold

then adult ticktet = $300 - x$

and $400x + 500(300 - x) = 144400$

M1(Forming the equation)

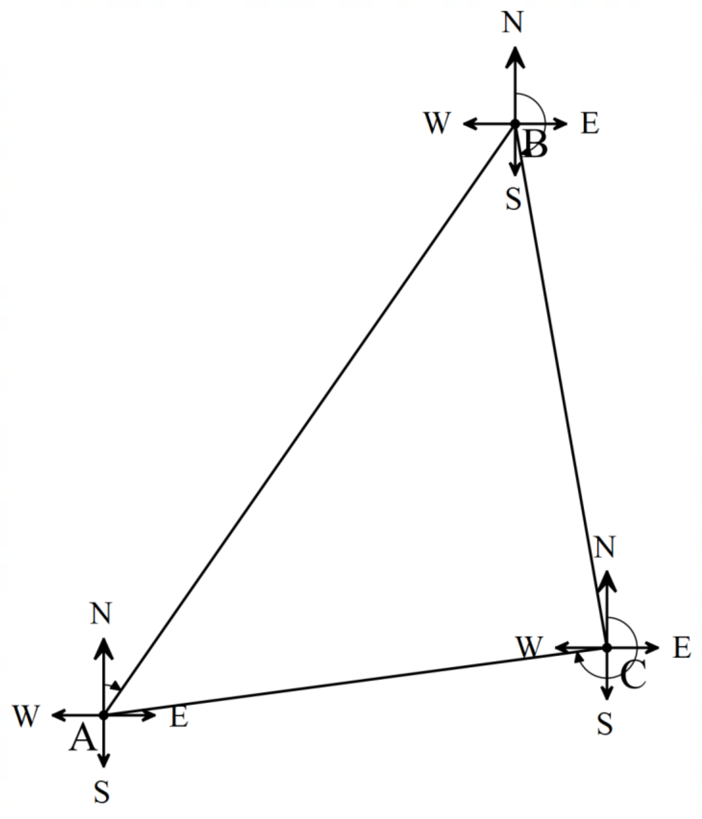
$$400x + 150000 - 500x = 144400$$
$$x = 56$$

M1 (solving the equation)

A1

(3 Marks)

5. A ship sails from point A on a bearing of 035^0 for 9.5km to point B. At B the ship alters course and sails for 7km on a bearing of 170^0 to point C. Use a scale drawing to find the distance and bearing of A from C. (4 marks)



B1 (Point B correctly located)
B1 (Point C correctly located

Bearing of A from C is 262°

Distance of A from C is 6.7km

B1 (Correct bearing of A from C)
B1(Correct distance of C from A

6. Given that $\sin x = \frac{2}{5}$, find the exact value of $\cos^2 x$ (2 marks)

$ALTERNATIVE\ 1$

$\cos x = \frac{\sqrt{21}}{5}$ **B1**

$\cos^2 x = \frac{21}{25} = 0.84$ **B1**

$ALTERNATIVE\ 2$ **Or**

$x = \sin^{-1}\left(\frac{2}{5}\right) = 23.57817848$

$\cos\left(\sin^{-1}\left(\frac{2}{5}\right)\right) = 0.916515139$ **B1(for cos x)**

$\cos^2 x = 0.84$ **B1**

7. A train whose length is 86m is travelling at a speed of 28km/h in the same direction as a truck whose length is 10m. if the truck takes 10.8 s to completely overtake the train, calculate the speed of the truck in km/h. (3marks)

$$\frac{86+10}{x-\left(28\times\frac{5}{18}\right)} = 10.8$$ **M1**

$$\frac{96}{x-\frac{70}{9}} = 10.8$$

$$x = 16\tfrac{2}{3}$$

$$x = 16\tfrac{2}{3}\times\frac{18}{5}$$ **M1 (conversion)**

$$x = 60\mathrm{km}h^{-1}$$ **A1**

8. The displacement S metres of a particle moving in a straight line after t seconds is given by $S = 2t^2 + 3t - 6$. Find the velocity of the particle during the fourth second. (3 marks)

$v = \frac{ds}{dt} = 4t + 3$ **M1**

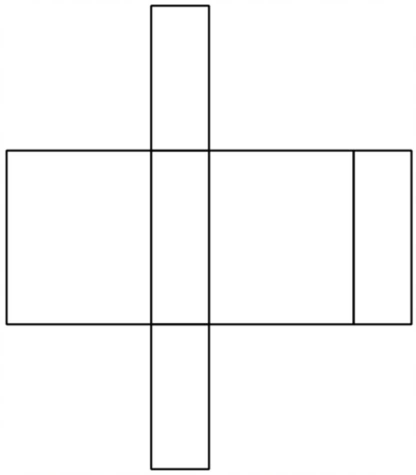
at $t = 3$

$v = 4(3) + 3 = 15$

M1

$v = 5ms^{-1}$ **A1**

9. The diagram below shows part of the net of a cuboid. Complete the net. (2 marks)



10. The surface areas of two similar solids are 352cm^2 and 792 cm^2 respectively. If the smaller solid has a mass of 1408g, find the mass of the larger solid. (3marks)

$$ASF = \frac{792}{352} = \frac{9}{4}$$ **M1**

$$\text{LSF} = \sqrt{\frac{9}{4}} = \frac{3}{2}$$

$$VSF = \left(\frac{3}{2}\right)^3 = \frac{27}{8}$$

$$\frac{27}{8} = \frac{x}{1408}$$ **M1**

$$x = 4752g$$ **A1**

11. David paid rent using $\frac{1}{10}$ of his salary. He used $\frac{1}{2}$ of the remaining amount to make down payment for a plot. He gave his mother Ksh. 2 500 and paid school fee balance for his son of Ksh.7 500. He then saved Sh. 12,500. How much was the down payment for the plot? (4marks)

Let the salary be x

$$\text{Rent} = \frac{1}{10}x$$

$$\text{Remainder } \frac{9}{10}x$$

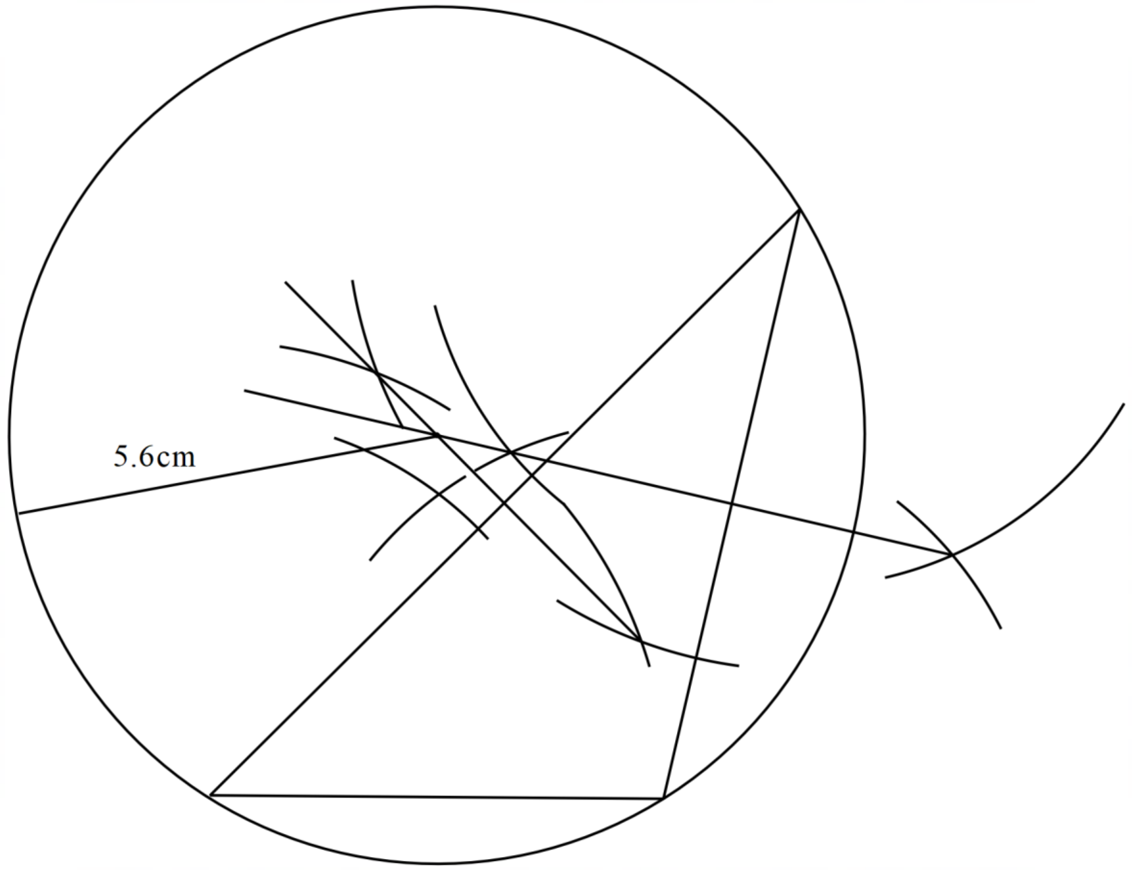
$$\text{Down payment} = \frac{1}{2} \times \frac{9}{10}x = \frac{9}{20}x$$ **M1(expression for downpayment)**

$$x - \left(\frac{1}{10}x + \frac{9}{20}x\right) = 2500 + 7500 + 12500$$ **M1(forming the equation)**

$$x = 50000$$ **A1**

$$\text{Down payment} = \frac{9}{20} \times 50000 = 22500$$ **B1**

12. Using a ruler and a pair compass only, construct a triangle ABC such that AB=6cm, BC= 8cm and AC= 11cm. draw a circle passing through the vertices of the triangle. Measure the radius of the circle. (4 marks)



B1 (triangle ABC drawn)

B1 (for center determined)

B1(circle drawn)

B1(for radius = 5.6cm)

13. An interior angle of a regular polygon is five times its exterior. Find the number of sides of the polygon. (3marks)

$$x + 5x = 180$$

$$x = 30$$ **B1 (for the exterior angle)**

$$\text{number of sides} = \frac{360}{30}$$

$$= 12$$ **B1**

14. A rectangle whose length is 9cm longer than its width has an area of $36cm^2$. If the width is x cm, form an equation in x and solve it to find the dimensions of the rectangle (3marks)

$$x(x + 9) = 36$$

$$x^2 + 9x - 36 = 0$$ **M1(forming the equations**

$$x = \frac{-9 \pm \sqrt{(-9)^2 - 4(1)(-36)}}{2(1)}$$ **M1(solving**
Accept alternatives ie (x+12)(x-3)=0

$$x = 3 \text{ or } -12$$

$$\text{length} = 12\text{cm and widt } h = 3\text{cm}$$ **A1**

15. Solve for $3-3x \le x+7 \le 9-x$ hence state the integral values of x (3marks)

$3-3x \le x+7$

$3-7 \le x+3x$

$-4 \le 4x$

$-1 \le x$

$x+7 \le 9-x$ **B1 (for either correct inequality**

$2x \le 2$

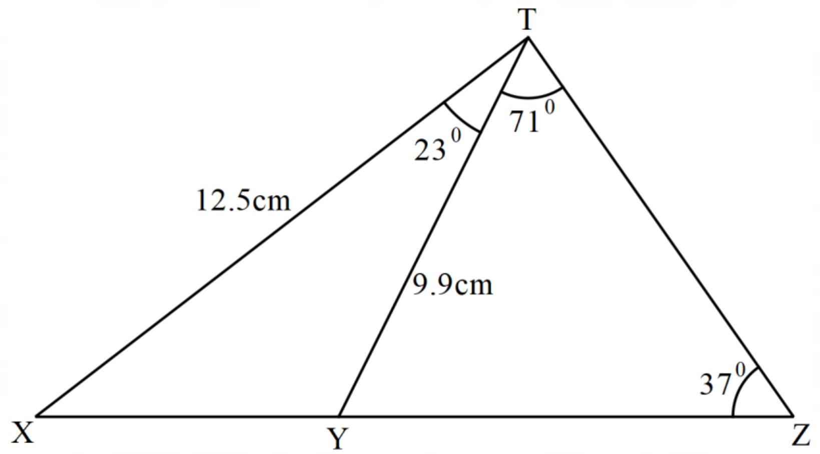
$x \le 1$

$-1 \le x \le 1$

Integers are -1, 0 and 1

B1(for the compound statement)
B1 (for the integers)

16. In triangle TXZ below, TX = 12.5cm and angle TZX = 37^0. Y is a point on the line XZ such that TY = 9.9cm, angle XTY = 23^0 and angle YTZ = 71^0.



Calculate to 1 decimal place:

a) the length of side XY (2 marks)

$(XY)^2 = 12.5^2 + 9.9^2 - 2 \times 12.5 \times 9.9\cos 23°$ **M1**

$XY = \sqrt{35.64}$

$XY = 6.0\text{cm}$ **A1**

b) The length of side TZ (2mks)

$\angle TYZ = 180 - (71 + 37) = 72°$

$\therefore \dfrac{TZ}{\sin 72} = \dfrac{9.9}{\sin 37}$ **M1**

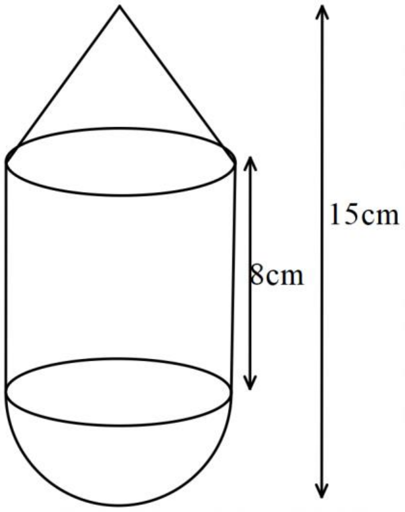
$TZ = \dfrac{9.9\sin 72}{\sin 37}$

$TZ = 15.6\text{cm}$ **A1**

SECTION II (50 MARKS)

Answer only **five** questions from this section in the spaces provided.

17. The figure below shows a model of a storage tank is made up of a conical top, a hemispherical bottom and the middle part is cylindrical. The total height of the model is 15cm, diameter of the cone, cylinder and the hemisphere is 6cm and the height of the cylindrical part is 8cm.



Calculate:

a) the total external surface area of the model in terms of π (5 marks)

$$\text{Area} = 2\pi(3)^2 + \pi \times 6 \times 8 + \pi \times 3 \times 5$$
$$= 81\pi \text{ cm}^2$$

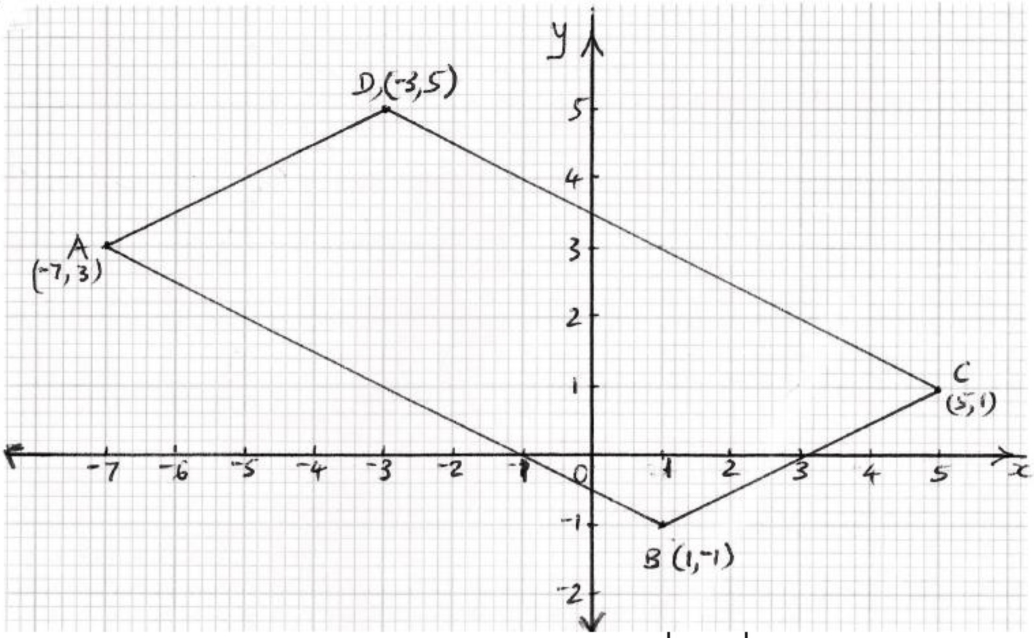
M1 (Area of hemisphere)
M1 (Area of curved surface of cylinder
B1(for 5 slant height of cone
M1 (Area of curved surface of cone)
A1

b) the total volume of the model in cm^3 correct to 2 significant figures, (5 marks)

$$\text{Vol} = \frac{2}{3}\pi \times 3^3 + \pi \times 3^2 \times 8 + \frac{1}{3}\pi \times 3^2 \times 4$$
$$= 320 \text{ cm}^3$$

M1 (vol of hemisphere)
M1 (vol of cylinder
M1 (vol surface of cone)
M1 (Adding the volumes)
A1

18.



18.	(a)	B1 B1	plotting vertices A, B and C. identifying vertex D (-3, 5) and competing parallelogram.
	(b) (i) grad AB $= \dfrac{3--1}{-7-1}$	M1	
	$= -\dfrac{1}{2}$	A1	
	(ii) $\dfrac{y-3}{x--7} = -\dfrac{1}{2}$ or $\dfrac{y--1}{x-1} = -\dfrac{1}{2}$	M1	
	$y = -\dfrac{1}{2}x - \dfrac{7}{2} + 3$ or $y = -\dfrac{1}{2}x + \dfrac{1}{2} - 1$		
	$y = -\dfrac{1}{2}x - \dfrac{1}{2}$	A1	
	(c) (i) Let grad L be m		
	$\therefore -\dfrac{1}{2}\text{m} = -1 \Longrightarrow \text{m} = 2$	B1	
	equation of line $\dfrac{y-3}{x-1} = 2$	M1	
	$y - 2x = 1$	A1	
	(ii) y - intercept: when $x = 0$		
	$\text{y} = 2 \times 0 + 1 = 1$		
	$\therefore$ co-ordinates $(0, 1)$	B1	
		10	

19. The table below shows the age in years of workers in a factory

Age x	x	f	fx
15 – 20	17.5	4	70
20 – 25	22.5	10	225
25 – 30	27.5	6	165
30 – 40	35	22	770
40 - 60	50	8	400
		$\sum f$ =50	$\sum fx$ =1630

a) Calculate the estimate of:

(i) The mean age of the workers (3 marks)

$$\text{Mean} = \frac{1630}{50}$$ **M1 (fx column)**

M1

$$= 32.6$$ **A1**

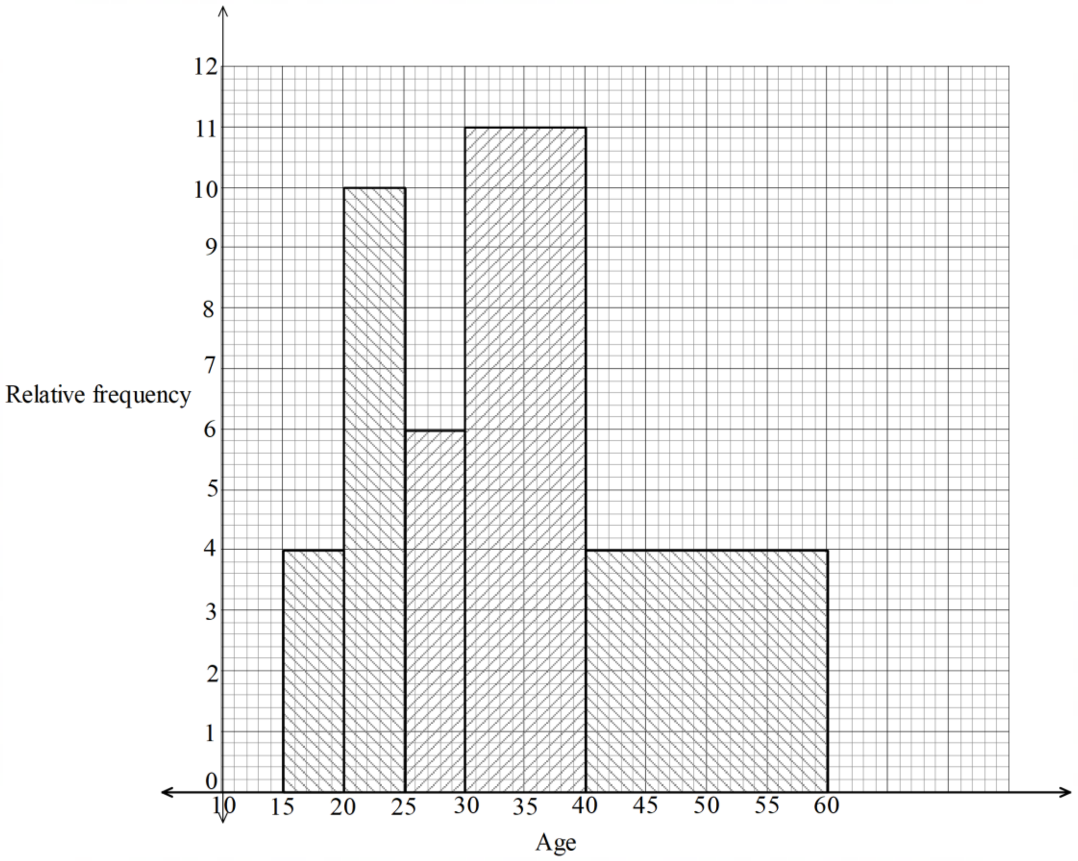
(ii) The median age of the workers (3 marks)

Cf 4, 14, 20, 42, 50 **B1 (for cf)**

$$\text{Median} = 30 + \frac{5}{22} \times 10$$ **M1**

$$= 32\tfrac{3}{11} = 32.27$$ **A1**

b) (i) Draw a histogram to represent the data (3 marks)



B3(All the rectangles correct)

B2 (At least 4 rectangles correct)

B1(At least 3 rectangles correct)

NB Frequency Density can be used too.

(ii) Use the histogram to determine the number of workers who are aged 23 and below years. (1 mark)

$$23 \textit{year} \text{ and below} = 4 + \frac{30}{50} \times 10 = 10 \text{ people}$$ **B1**

20. a) Given $\mathbf{A} = \begin{pmatrix} -2 & 4 \\ 1 & 0 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 0 & 1 \\ p & q \end{pmatrix}$, and that **AB** = **I**, find the value of p and q. (4 marks)

$$\begin{pmatrix} -2 & 4 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ p & q \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$ **M1 (correct equation with I)**

$$\begin{pmatrix} 4p & -2+4q \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$ **M1 Correct LHS)**

therefore:

$$4p = 1$$

$$p = \frac{1}{4}$$ **A1**

and

$$-2 + 4q = 0$$

$$4q = 2$$

$$q = \frac{1}{2}$$ **B1**

b) Find $\mathbf{P}^{-1}$, the inverse of the matrix $\mathbf{P} = \begin{pmatrix} 5 & 3 \\ 2 & 7 \end{pmatrix}$.

Hence determine the coordinates of the point of intersection of the lines:
5x + 3y = 21 and 2x + 7y = 20 (6 marks)

$$\boldsymbol{P}^{-1} = \frac{1}{29}\begin{pmatrix} 7 & -3 \\ -2 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{7}{29} & -\frac{3}{29} \\ -\frac{2}{29} & \frac{5}{29} \end{pmatrix}$$ **B1 (for determinant**
B1 (for $\mathbf{P}^{-1}$ correct)

$$\begin{pmatrix} 5 & 3 \\ 2 & 7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 21 \\ 20 \end{pmatrix}$$ **M1(Matrix equation)**

$$\begin{pmatrix} \frac{7}{29} & -\frac{3}{29} \\ -\frac{2}{29} & \frac{5}{29} \end{pmatrix}\begin{pmatrix} 5 & 3 \\ 2 & 7 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{7}{29} & -\frac{3}{29} \\ -\frac{2}{29} & \frac{5}{29} \end{pmatrix}\begin{pmatrix} 21 \\ 20 \end{pmatrix}$$ **M1**

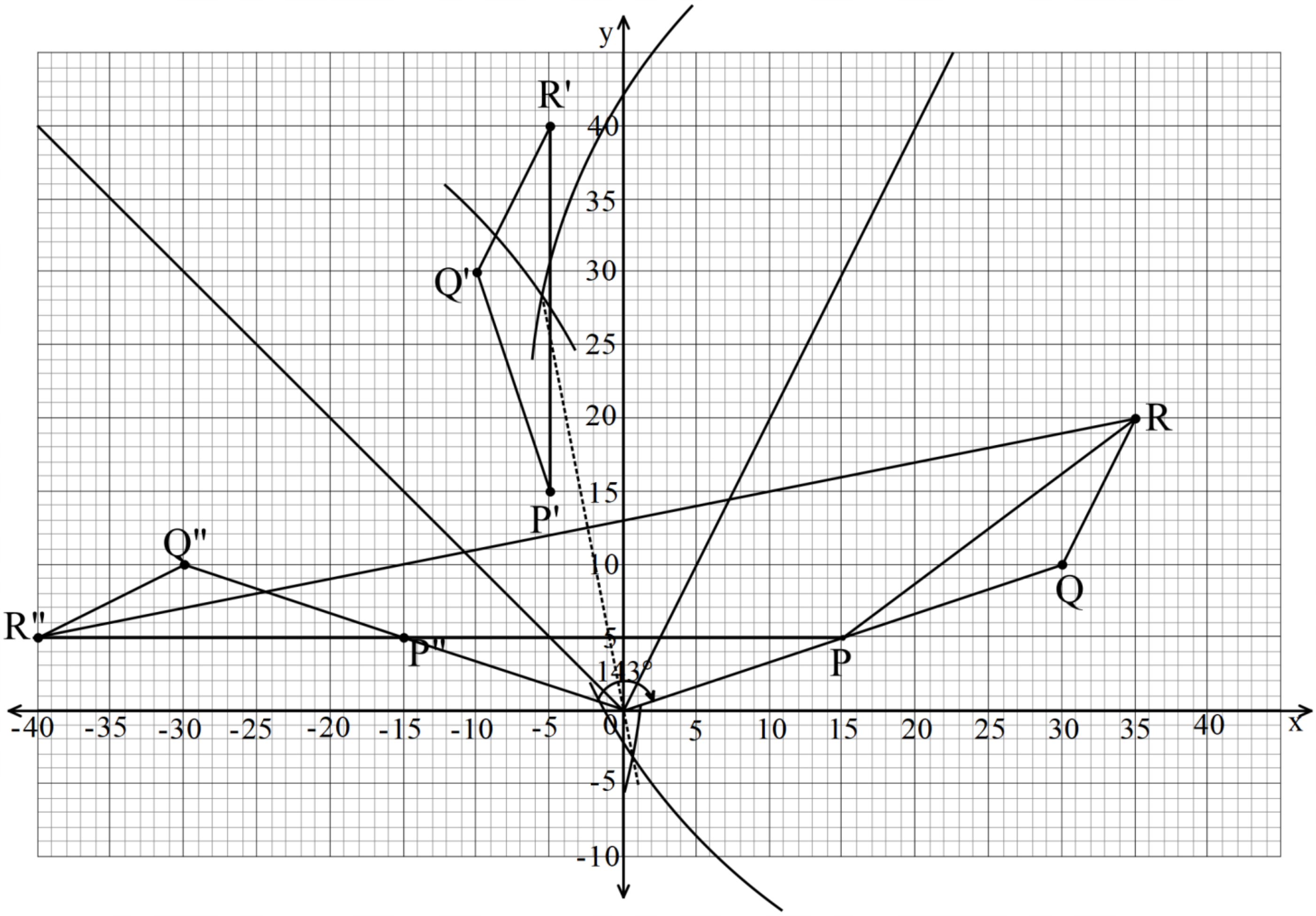
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$ **A1**

$x = 3$ and $y = 2$

Point of intersection (3,2) **B1 (for the coordinate)**

21. On the grid provided, Using a scale of 1cm to represent 5 units on each axis and taking values of x from -40 to 40 and values of y from -10 to 40.

a) Draw triangle PQR with vertices P(15, 5), Q(30, 10) and R(35, 20) (2 marks)



b) Draw triangle P'Q'R', the image of triangle PQR under reflection in the line y = 2x. (3 marks)

c) Draw triangle P"Q"R", the image of triangle P'Q'R' under a reflection in the line y + x = 0. (2 marks)

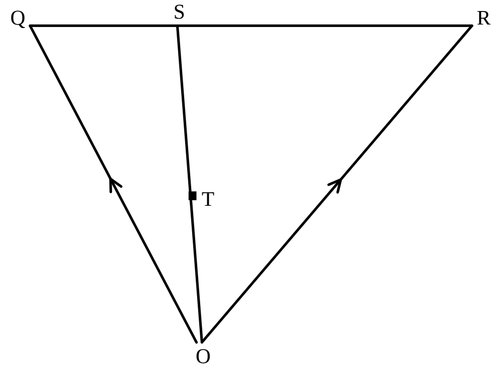
d) Determine by construction, the centre and the angle of rotation that maps triangle P"Q"R" onto triangle PQR. (3 marks)

Centre of rotation (0,0)

Angle of rotation -143^0 or 217^0

a) B1 correct scale
B1 triangle PQR drawn and correctly labeled
b) B1 line y = 2x drawn
B1 any of the three points P', Q', R' located correctly
B1 correct triangle P'Q'R' drawn and labelled
c) B1 line y+x = 0 drawn
B1 correct triangle P"Q"R" drawn and labelled
d) B1 Perpendicular bisector of line RR" constructed
B1 centre of rotation
Bi Angle of rotation

22. In the figure below S divides QR in the ratio 1:2, T divides OS in the ratio 3:2, **OR** = **r** and **OQ** = **q**.



a) Write in terms of **q** and **r**:

i) **RQ** (1 mark)

$-r + q$ **B1**

ii) **OS** (2 marks)

$q + \frac{1}{3}(-q + r)$ **M1**

$\frac{2}{3}q + \frac{1}{3}r$ **A1**

iii) **RT** (2 marks)

$-r + \frac{3}{5}\left(\frac{2}{3}q + \frac{1}{3}r\right)$ **M1**

$\frac{2}{5}q - \frac{4}{5}r$ **A1**

b) i) If L is the midpoint of line OQ, show that the points R, T and L are collinear. (4 marks)

$RL = -r + \frac{1}{2}q = \frac{1}{2}q - r$ **M1 (For RL or any other relevant vector)**

Let $RL = mRT$

$\frac{1}{2}q - r = \frac{2}{5}mq - \frac{4}{5}mr$ **M1(looking for the scalar)**

$\frac{4}{5}m = 1$ and $\frac{1}{2} = \frac{2}{5}m$

$m = \frac{5}{4}$ $\Rightarrow$ $m = \frac{5}{4}$

$\therefore$ $RL = \frac{5}{4}RT$ **A1**

$\Rightarrow$ RL is parallel to RT

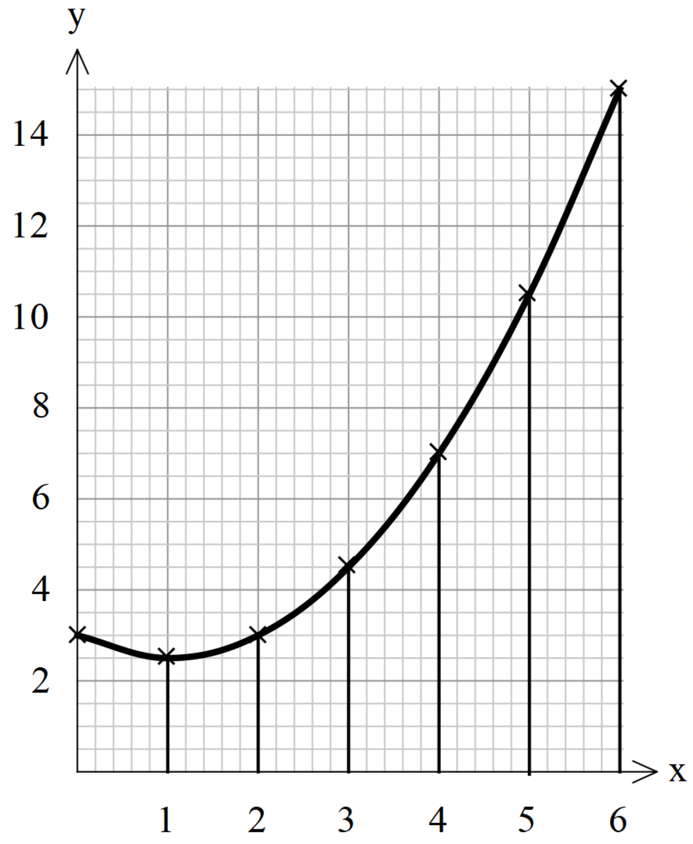
Since R is a common point, R, T and L are collinear **B1(mentioning parallel and picking out the common point**

ii) Hence find the ratio of RT:TL (1 mark)

4 : 1 **B1**

23.

a) On the grid provided, draw the graph of the function $y=\frac{1}{2}x^2-x+3$ for $0\le x\le 6$. (3 marks)



S1(suitable scale)

P1

C1(smooth curve)

b) Use the graph and the trapezium rule, to approximate the area under the curve between x = 1, x=6 and the x axis using 6 ordinates. (3 marks)

$A = \frac{1}{2} \times 1[2.5 + 15 + 2(3 + 4.5 + 7 + 10.5)]$ **B1(for the correct ordinates)** **M1**

$A = 33.75$ sq Units **A1**

c) Calculate the mid-ordinates for 5 strips between x = 1 and x = 6 and hence use the mid-ordinate rule to approximate the area under the curve between x = 1, x=6 and the x axis. (3marks)

x	1.5	2.5	3.5	4.5	5.5
y(mid ordinate)	2.625	3.625	5.625	8.625	12.625

B1(for the correct ordinates)

Area = $1[2.625 + 3.625 + 5.625 + 8.625 + 12.625]$ **M1**

$= 33.125$ sq units **A1**

d) Determine the difference in area between the trapezium rule and the mid-ordinate rule estimates (1 mark)

$33.75 - 33.125 = 0.625$ **B1**

24. The equation of a curve is $y = 2x^3 - 9x^2 + 12x - 9$.

a) The gradient of the curve when x = 2. (3 marks)

$\frac{dy}{dx} = 6x^2 - 18x + 12$ **M1**

at $x = 3$

$6(3)^2 - 18(3) + 12$ **M1**

gradient $= 12$ **A1**

b) i) The turning points of the curve. (3 marks)

$\frac{dy}{dx} = 6x^2 - 18x + 12$

At stationary point $\frac{dy}{dx} = 0$

$\therefore 6x^2 - 18x + 12 = 0$ **M1**

$(6x - 12)(x - 1) = 0$ **A1**

$\Rightarrow x = 1$ or 2

At $x = 1$

$y = 2(1)^3 - 9(1)^2 + 12(1) - 9 = -4$

Point $(1, -4)$

at $x = 2$

$y = 2(2)^3 - 9(2)^2 + 12(2) - 9 = -5$

Point $(2, -5)$ **B1(for both points)**

ii) The nature of the turning point b(i) above. (2 marks)

Second derivative $\frac{d^2y}{dx^2} = 12x - 18$

at the point $(1, -4)$

$12(1) - 18 = -6$

Since the value of the second derivative is negative the point is a maximum **B1**

At the point $(2, -5)$

$12(2) - 18 = 6$

Since the value of the second derivative is positive the point is a minimum **B1**

c) Sketch the curve. (2 marks)

B1 for the y intercept
B1 for the sketch